Examinee's No. ____________________

Department of Nuclear Engineering
and Management, The University of Tokyo
2009 Entrance Examination for the Master Course

Mathematics

September 1, 2008
13:00～15:30

Notice

1. Do not open this booklet until the start of the examination.

2. Write your examinee's number on top of this sheet. Do not write your examinee's number or your name anywhere else in this booklet.

3. Write the outline of your solution below the question and put only the answer in the box.

4. Sheets for calculation are distributed separately.

5. There are 20 questions in total. Choose and answer 15 questions out of 20.
Check the number of the questions you have selected in the bottom row.
Notice that you cannot answer more than 15 questions.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Q1.

Obtain A^n of the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix},$$

where n is a positive integer, i is the imaginary unit, and

$$\omega = \frac{-1+\sqrt{3}i}{2}.$$

Q2.

When 9 letters, AAAABBCCC are aligned randomly, what is the probability that the symmetrical alignments, such as AABCCCBAA are generated?

Q3.

Obtain all combinations of the positive integer numbers l, m, and n that satisfy the following relational expression:

$$lmn = 2l + m + n, \quad l \geq m \geq n.$$

Q4.

One black ball and three white balls (a total of four balls) are included in each X and Y box. One ball is exchanged between these boxes in one trial. Obtain the probability of a specific state in which each box has one black ball and three white balls after the N^{th} trial.

Q5.

Consider a sphere of radius r. When the section of the sphere intersected by the $x-y$ plane $(z=0)$ is a circle of radius a, the section of the sphere intersected by the $y-z$ plane $(x=0)$ is a circle of radius b, and the section of the sphere intersected by the $z-x$ plane $(y=0)$ is a circle of radius c, find the distance between the origin of the coordinates and the center of the sphere.

Q6.

Evaluate

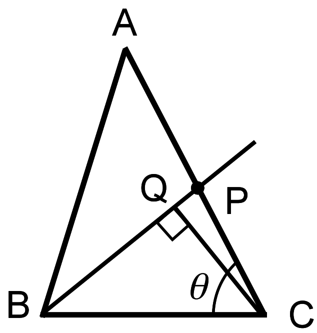
$$\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx.$$

Q7.

Find the total area of the regions bounded by functions e^{-x} and $e^{-x}\sin x$ in the semi-infinite interval $0 \le x < +\infty$.

Q8.

Consider a triangle ABC where $AC = 4$, $BC = 3$, $\angle ACB = \theta$, $\overrightarrow{CA} = \vec{a}$, and $\overrightarrow{CB} = \vec{b}$. The point P is the midpoint of the side AC, and a perpendicular line CQ is drawn from the point C to the straight line BP. Express the $\overrightarrow{CQ}$ using θ, $\vec{a}$, and $\vec{b}$.



Q9.

Vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are shown in Fig. 1. Parallelepiped V_1 is constructed using these vectors, as shown in Fig. 2. Then hexahedron V_2 is constructed inside V_1 using the center points of faces and two vertices of V_1, as shown in Fig.3. When $\vec{a} = (3, 1, 0)$, $\vec{b} = (2, 3, 1)$, and $\vec{c} = (1, 1, 3)$, calculate the volume of V_2.

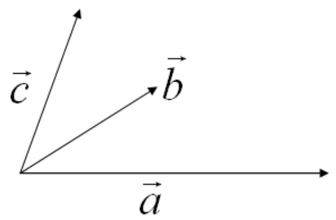


Fig. 1

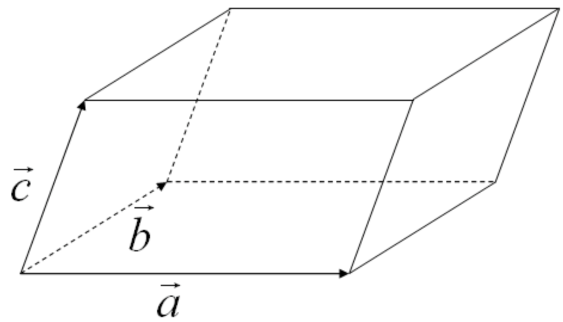


Fig. 2

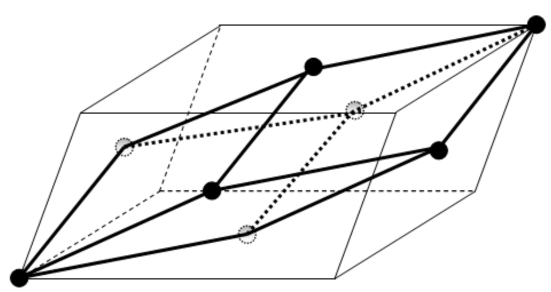


Fig. 3

Q10.

Obtain the area of the circle that is formed by the intersection of two spherical surfaces:

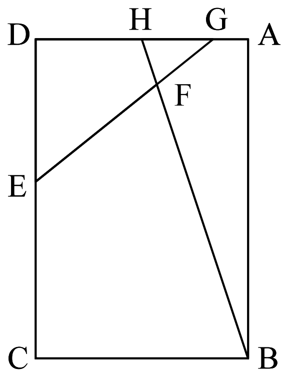
$$S_1: x^2 + y^2 + z^2 = 1,$$
$$S_2: x^2 + y^2 + z^2 + 4x - 4y + 2z + 1 = 0.$$

Q11.

A point P is chosen at random on the circle $x^2 + y^2 = 1$. The variable X denotes the distance of P from $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Find the mean of X.

Q12.

Find the area of the quadrangle EFBC, where ABCD is a rectangle and AG = 1, GH = 2, HD = 3, DE = 4, EC = 5.



Q13.

Find A and B when numbers 1 to 16 are put in each cell of the 4×4 matrix below such that the sums of 4 numbers in all directions (row, column, or diagonal) are identical.

4		15	B
5	11		
	7		12
A			13

Q14.

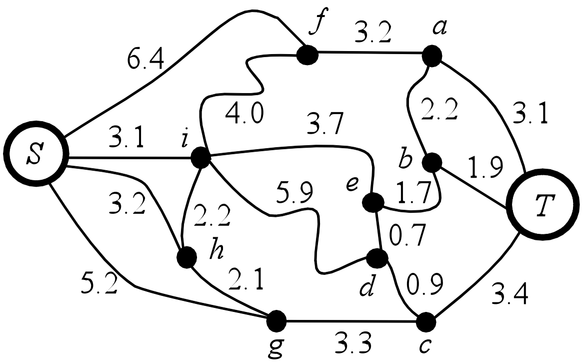
Evaluate

$$\left(\sqrt{3}i - 1\right)^{10} + \left(-\sqrt{3}i - 1\right)^{10},$$

where i is the imaginary unit.

Q15.

In the graph shown below, find the shortest path between S and T, where the numbers beside the branches indicate the length of the corresponding branches.



Q16.

Find the value of Ａ in the following calculation.

$$
\begin{array}{cccccc}
 & & & & \square & \square \\
 & & \times & \square & \square & 8 \\
\hline
 & & & \square & \square & \square \\
 & & & 0 & 0 & \\
 & & \square & \square & & \\
\hline
 & 1 & \square & 6 & \text{A} & \square
\end{array}
$$

Q17.

P and Q are defined as:

$$P = \sqrt{2 + 3\sqrt{2 + 3\sqrt{2 + 3\sqrt{2 + \cdots}}}},$$

$$Q = a + \cfrac{2}{a + \cfrac{2}{a + \cfrac{2}{a + \cdots}}}.$$

Suppose P and Q converge, and $a > 0$. When $P = Q$, find a.

Q18.

A fraction, which includes 4 numbers multiplied in the numerator and 3 numbers multiplied in the denominator, has a value of 1:

$$\frac{\square\cdot\square\cdot\square\cdot\square}{\square\cdot\square\cdot\square}=1.$$

Use 7 numbers (2, 4, 8, 16, 32, 64, 128) for the numerator and the denominator. Find all combinations of 4 numbers in the numerator and 3 numbers in the denominator.

Q19.

How many natural numbers are there below 1000 that are multiples of 3 or that contain 3 in any digit of the number?

Q20.

Following two expressions represent words written in code:

(49, 75, 113, 126, 129): key 37 = labor,
(71, 45, 53, 67, 112, 82): key 31 = invest.

Break the code and find the word for:

(106, 112, 77, 107, 92, 71): key 29 = ???