Examinee Number _____

2017 Entrance Examination
Department of Systems Innovation /
Department of Nuclear Engineering and Management /
Department of Technology Management for Innovation,
Graduate School of Engineering, The University of Tokyo

Mathematical Problems Designed to Test Ability to Think Logically

Booklet of Problems and Answer Sheets

Monday, August 29, 2016 13:00 - 15:30

Documents distributed:

1. Booklet of Problems and Answer Sheets for 2017 Entrance Examination, Mathematical Problems Designed to Test Ability to Think Logically (this booklet)
2. Problem booklet for 2017 The Graduate School Entrance Examination, Mathematics
3. Answer sheets for 2017 The Graduate School Entrance Examination, Mathematics
4. Calculation sheets

General instructions:

- ○ Do not open any booklets until the start of the examination is announced.
- ○ Confirm that all documents above are correctly distributed. Notify your proctor if you find any missing items.
- ○ Notify your proctor if you find any printing or production errors.
- ○ Write your examinee number in the designated places of Document 1 (this booklet), Document 2 (2017 The Graduate School Entrance Examination, Mathematics), and Document 3 (Answer sheets), respectively.
- ○ Do not take any items distributed with you after the examination.
- ○ **Answer four questions out of the six given in this booklet.** Write your answer below each question statement, including your solution process. Circle the questions you selected in the following table. **You are not allowed to choose more than four questions.**

Selected questions	Q. 1	Q. 2	Q. 3	Q. 4	Q. 5	Q. 6

- ○ **Answer two problems out of the six given in Document 2** (2017 The Graduate School Entrance Examination, Mathematics) **on the answer sheets** (Document 3)**. You are not allowed to answer more than two problems.**

Q. 1

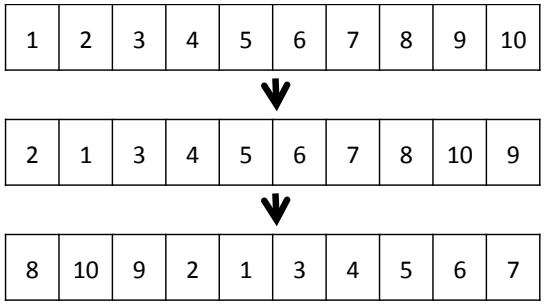
Ten cards, each of which has a different number, are arranged in a row on the table. The procedure consisting of the following three operations will be applied:

(a) The leftmost card is switched with the next card.

(b) The rightmost card is switched with the next card.

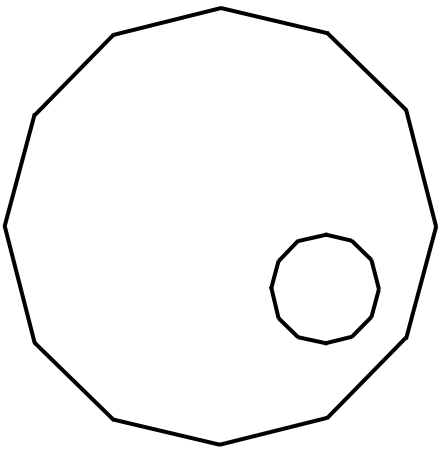
(c) The three cards from the right edge are moved to the left edge, keeping its sequence.

For example, if this procedure is applied once, the card sequence changes as shown in the following figure. How many times of this procedure should be repeated until the original sequence first reappears?



Q. 2

In the figure below, the outer regular dodecagon is inscribed in a circle of four in radius, and the inner one in a circle of one in radius. Obtain the area of the region bounded between these two regular dodecagons.



Q. 3

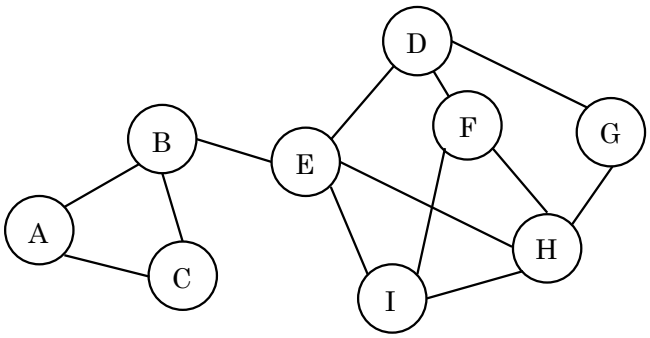
Five pirates and one monkey were shipwrecked and drifted down to an island. On that day, they gathered nuts to eat. At night, a pirate woke up and gave one nut to the monkey. Then he divided the remaining nuts equally to five piles and hided one pile for himself. Having merged the remaining four piles, he fell asleep again as if nothing had happened. Further late night, the other four pirates did the same thing sequentially, that is, each of them gave one nut to the monkey and hided one fifth of the remaining nuts. Next morning, they gave one nut to the monkey and divided the remaining nuts in five equal shares. Obtain the minimum number of nuts the pirates gathered. Note that the nuts were always divided equally without a remainder.

Q. 4

Concerning the network shown in the figure below, answer the following questions.

(1) How many paths exist that visit each node once?

(2) Suppose choosing some nodes from this network so that the set of edges connected to any of the chosen nodes covers all the edges of the network. Find all the combinations that have the minimum number of chosen nodes.



Q. 5

Each element of a matrix with k rows and m columns is $+1$ or -1. The product of all elements in a row (called "row product") is -1 for any row, and that in a column (called "column product") is -1 for any column. In this case, answer the following questions.

(1) Let r be the product of all row products, and let c be the product of all column products. Find $r \times c$.

(2) Prove that the matrix satisfying the above conditions does not exist when $k + m$ is an odd number.

(3) Find the number of matrices satisfying the above conditions when $k + m$ is an even number.

Q. 6

Variable a, b, c or d is a logical variable. Answer the following questions.

(1) Find all the combinations of truth values for a, b and c that make all the following logical formulae True.

$\neg a \vee b \vee c$

$a \vee \neg b \vee \neg c$

$\neg a \vee b \vee \neg c$

$\neg a \vee \neg b \vee c$

(2) How many combinations of truth values for a, b, c and d make all the following logical formulae True?

$\neg a \vee b \vee \neg c \vee \neg d$

$\neg a \vee b \vee c \vee \neg d$

$a \vee \neg b \vee c \vee d$

$a \vee \neg b \vee \neg c \vee \neg d$

$\neg a \vee \neg b \vee c \vee \neg d$